

## Optical Specification of Time-to-Passage: Observers' Sensitivity to Global Tau

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Despite its general mathematical formulation, most empirical work on the visual perception of tau (defined as a quantity divided by its temporal derivative) has focused on the case of direct approach, with tau defined as image angle/rate of expansion. Empirical investigators tend to generalize image size analyses to off-axis approaches. However, this generalization is inappropriate for all but a few classes of objects. After mathematically reestablishing the appropriate optical cues specifying time to passage for noncollision cases, we report a series of studies in which we examined observers' sensitivities to this information in both relative- and absolute-judgment paradigms. In general, we found observers' judgments to be accurate and robust.

When an object and an observer move relative to each other, there is optical information that specifies the time until closest approach. Most of the theoretical and empirical research on this problem has dealt with the situation in which the object and observer are on a collision course (Lee, 1976; Schiff & Detwiler, 1979). However, in much skilled activity, the goal is not to determine when an object will collide, but rather to coordinate behavior based on when an object will be passed. For example, in low-level flight, helicopter pilots orchestrate their behavior on the basis of the time until the next critical landmark is passed. Thus, it is important to know what visual information is potentially available to specify time to arrival, not only for objects on the direct flight track,<sup>1</sup> but also for those off track.

For direct collisions, a source of optical information specifying time-to-collision (TTC) is the instantaneous rate of change in the visual extent of the object (or of discriminable areas, contours, or texture elements on the object) relative to the extent of the instantaneous image (Hoyle, 1957; Lee, 1976). This estimate of TTC, referred to as tau, is then defined as follows:

$$TTC = \tau = \phi_r / (\delta\phi/\delta t) \quad (1)$$

where  $\phi_r$  is the angular extent of the object image at time  $t$  and  $\delta\phi/\delta t$  is the instantaneous rate of expansion of that visual angle.<sup>2</sup> The elegance of this formulation (and hence its

attraction as a potential source of visual information) is evident. TTC is, in principle, specified at every instant in the trajectory and can be derived purely from optical variables (without recovering velocity or distance information).<sup>3</sup> Studies on humans' and other species' ability to use this information have shown that, within certain temporal ranges, judged TTC is fairly well correlated with tau (Lee & Reddish, 1981; Lee, Young, Reddish, Lough, & Clayton, 1983; Schiff & Detwiler, 1979; Todd, 1981).

In fact, tau based on image extent is just one of three definitions put forth by Lee (1976, 1980; Lee & Young, 1985). Tresilian (1991) provided a useful taxonomy of the tau-type optical variables. In Tresilian's nomenclature, tau based on the reciprocal of the relative rate of dilation of an object image is termed *local tau, type 2* ( $\tau_L^{(2)}$ ). Tau based on the angle subtended by two designated points on an object (e.g., texture patches) divided by the angle's rate of expansion is *local tau, type 1* ( $\tau_L^{(1)}$ ). Note that for both types of local tau, the optical information can be derived local to the image of an approaching object (type 1 is the expansion de-

<sup>1</sup> We use the aerodynamic term "track" to refer to the observer's (and vehicle's) direction of motion relative to a stationary environment. Many authors describe this motion as the observer's "heading," but this is true only if there is no wind influencing observer/vehicle motion. Thus, heading and track are equivalent for most forms of terrestrial locomotion. In aviation, track is the resultant of the vehicle propulsion vector (whose direction is heading) and the wind vector.

<sup>2</sup> The formulations in this article use angular measurements, but an alternative geometry based on retinal (projection) distances can be used, as demonstrated by Todd (1981). Either derivation requires the use of a geometric approximation: for the angular derivation, the law of small angles (i.e.,  $\tan\theta \cong \theta$ ); for the projection derivation, a planar surface to approximate the retina.

<sup>3</sup> In fact, the elegance of image expansion rate for specifying time-to-collision independent of size and distance information appears to have been first noted by astrophysicist Fred Hoyle, who allowed one of his more clever characters to develop its proof in his 1957 science fiction novel, *The Black Cloud*. Later, perceptual psychologists, most notably David Lee, realized its potential significance as a visual cue.

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Portions of this article were presented at the 32nd Annual Meeting of the Psychonomic Society, in San Francisco, November 1991.

We thank Walter Johnson, John Perrone, Dennis Proffitt, and James Todd for their comments on a draft of this article; Terry Bahill, William Schiff, and two anonymous reviewers for their helpful suggestions; and Larry Beck of Sterling Software for his programming support.

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finer by two points, whereas type 2 is the expansion defined by the entire object image). Tresilian distinguished these two local taus from global tau ( $\tau_G$ ), which is instantiated in the global flow field. Global tau operates on the angular extent between an object and the observer's track vector ( $\theta$ ) divided by the rate of change of that extent ( $\delta\theta/\delta t$ ) (as shown in Figure 1 and derived in the Appendix). Global tau ( $\tau_G = \theta/[\delta\theta/\delta t]$ ) is the appropriate optical variable to use for time to passage (TTP; i.e., off-axis approaches).

The extraction of global tau is, of course, related to the more general problem of extracting spatial layout from optic flow. Global tau differs from more formal structure-from-optic-flow-pattern models (e.g., Koenderink, 1986; Longuet-Higgins & Prazdny, 1980; Nakayama & Loomis, 1974) in that a depth value is recovered for only a single element, and the depth is recovered in terms of an absolute temporal metric. Clearly, a comprehensive recovery of spatial layout and ego velocity from optic flow would also allow accurate TTP judgments. However, the strength of global tau is its ability to provide absolute temporal range information within a far simpler computational framework.

In some empirical studies (e.g., Schiff & Oldak, 1990), subjects are asked to judge approach times of objects that are not on direct collision trajectories. In describing the optical information available in these bypass situations, researchers usually suggest that these events approximate the collision geometry. Hence local tau should still describe the relevant temporal information (e.g., von Hofsten & Lee, 1985). However, examination of the geometry of noncollision trajectories reveals that the validity of local tau becomes severely compromised, as demonstrated in Figure 2, Table 1, and the

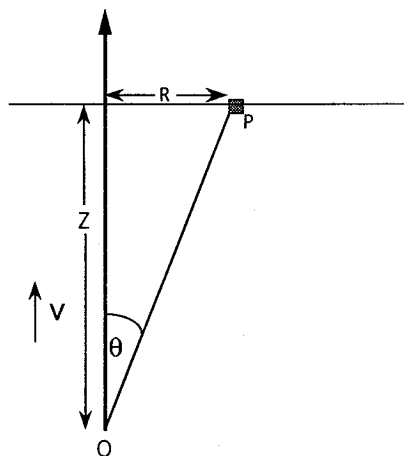


Figure 1. The projection geometry for a passage event, using Lee's (1976) notation. (Environmental variables [objects and distances] and velocities are indicated by capital letters. Distances are shown with double-headed arrows, velocity vectors with single-headed bold arrows and letters. An observer,  $O$ , moves along a track vector at a constant velocity,  $V = \delta Z/\delta t$ . An object,  $P$ , lies some distance,  $R$ , off the track vector. At time  $t$ , the observer is at some distance,  $Z$ , from the point of nearest approach to  $P$ , defined by passage through a plane orthogonal to the track vector that intersects the object. As delineated in the Appendix, time to contact (TTC) at  $t$  is defined by  $\theta/(\delta\theta/\delta t)$ .)

Appendix. In most analyses of the error introduced in these noncollision cases, the error is underestimated because the researchers assume no change in the image extent due to observer-relative rotation (Lee & Young, 1985; von Hofsten & Lee, 1985). That is, they assume that the image undergoes no shear during the approach. This assumption is valid if one is deriving the angular extent of spherical objects (e.g., a baseball or a cricket ball) or cylindrical objects aligned orthogonal to the track vector (e.g., a telephone pole along a highway). However, its validity is limited to a relatively small class of objects. Moreover, in all cases the approximation is compromised by the offset distance; that is, the image expansion is bounded by the angle subtended when the object is at its nearest approach. The impact of using local tau for inappropriate off-axis approaches is apparent from Table 1, which compares true TTP with local tau and global tau. Two aspects of Table 1 should be noted. First, in the range in which local tau is fairly accurate ( $\geq 4$  s), the rate of angular expansion is small ( $< 1^\circ/\text{s}$ ). Second, local tau breaks down completely at a distance of about 6 m in this example (the distance will vary as a function of offset and object dimensions). At that point, image expansion ceases (specifying an infinite local tau) and image contraction begins (leaving local tau undefined).

Although  $\tau_L^{(1)}$ ,  $\tau_L^{(2)}$ , and  $\tau_G$  can all be considered special cases of the same general mathematical formulation, they use different kinds of optical variables and therefore presuppose different perceptual competencies on the part of observers. As Tresilian (1991) pointed out, both types of local tau function on local image features—the rate of expansion of an image or the distance between two points on an object. Global tau operates on the distance between an object feature and the moving observer's track vector, the latter being instantiated in the observer's global flow field. Its utility presupposes two critical competencies:

1. The observer can determine his or her instantaneous track vector. Although the track vector is instantiated by the focus of expansion (FOE) of the flow field when the observer's gaze corresponds to his or her linear trajectory, other singularities exist for curvilinear trajectories or when eye/head movements occur. Here we consider only the simpler case in which FOE and track are correspondent. However, psychophysical evidence suggests that observers can extract track information from optical flow even in the presence of concomitant rotations (Cutting, Springer, Braren, & Johnson, 1992; Stone & Perrone, 1991; Warren & Hannon, 1990).

2. The perceptual system can determine the angle (or retinal distance) between an object and the track vector. Although this information is clearly specified, it has not been demonstrated that observers can accurately extract this parameter. Furthermore, competence may vary as a function of the angular extent (e.g., as the extent increases, accuracy may diminish).

Local tau and global tau also probably play different functional roles in orchestrating skilled actions. Local tau functions best for dealing with objects on a direct approach trajectory and within a fairly short temporal distance (e.g., ball catching). In fact, local tau information for objects outside the proximal temporal window is likely subthreshold. For

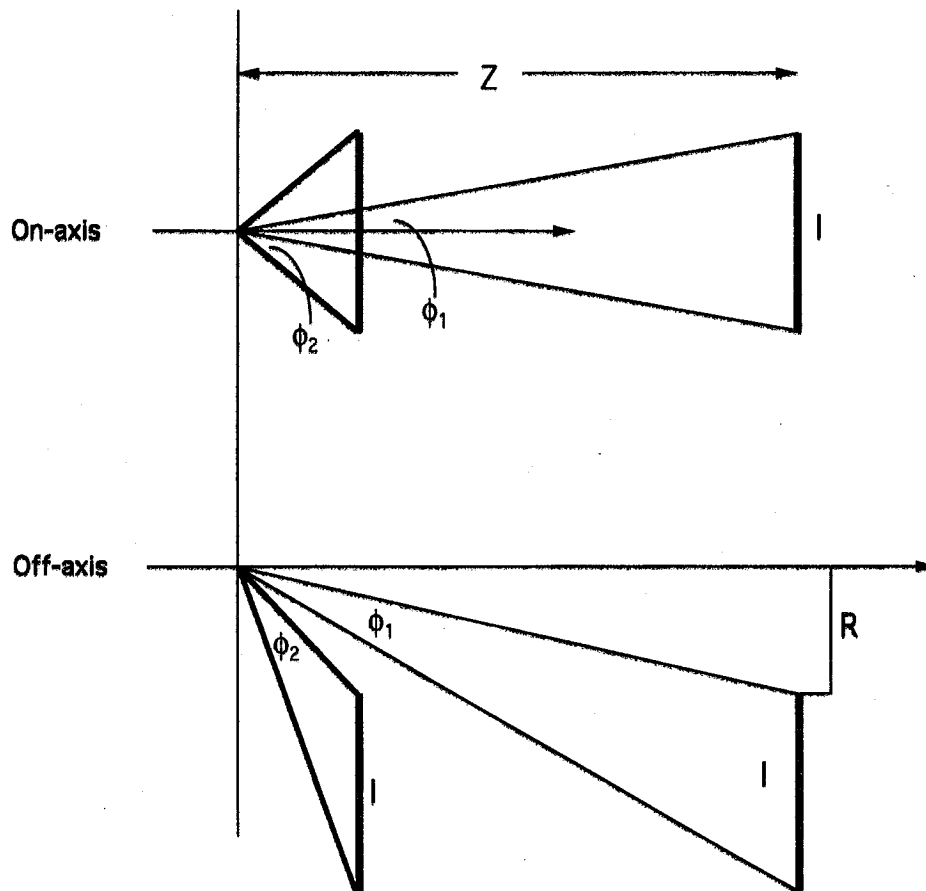


Figure 2. The fallacy of using local tau for an off-axis approach is demonstrated. (In the on-axis case,  $\phi$  is a function of  $I/Z$ , where  $I$  (object size) is time constant. For the off-axis case,  $\phi$  is a function of  $\cos(\theta)I/(Z^2 + R^2)^{1/2}$ , where  $\theta$  is the angle between the object and the track vector,  $Z$  is the distance from object to observer, and  $R$  is the offset of the object from the track vector. Because the numerator of the function is no longer time constant, the derivations shown in the Appendix are no longer valid, and  $TTC \neq \phi/(\delta\phi/\delta t)$ .)

example, an approaching object that presently subtends  $1^\circ$  of visual angle and is 10 s distant will have an expansion rate of only 6 min/s (Hill, 1980). Global tau would be much more salient for objects in such temporal windows. An object  $10^\circ$  off the track vector would have an angular velocity of  $1^\circ/s$  when 10 s distant, and an object  $20^\circ$  from track would move at  $2^\circ/s$ . Thus, global tau would, in theory, prove more useful for longer term orchestration of actions as one navigates through an environment.

A number of factors may affect global tau's actual utility. First, as mentioned earlier, it may be the case that the perceptual system cannot extract the angle between the track vector and the sight line to an object. Second, it has been suggested that the human visual system is specifically sensitive to the rate of change of size (Regan, Kaufman, & Lincoln, 1986). If this sensitivity is specific to image size, and no similar mechanism exists for changes in the extent between points in the visual field, then humans may be unable to exploit global tau information. In the animal literature, Wang and Frost (1992) recently reported finding a subpopulation of neurons in the nucleus rotundus of the pigeon brain

that responds selectively to objects approaching on a collision course. The response was markedly reduced when the object deviated  $5^\circ$  or more from a collision course. Thus, although the geometry of global tau is formally equivalent to that of local tau, it is conceivable that the human perceptual

Table 1

Comparison of Actual Time-to-Passage (TTP) With Values of Local Tau, Type 1 ( $\tau_L^{(1)}$ ) and Global Tau ( $\tau_G$ ) and Their Associated Optical Expansion Rates for the Off-Axis Approach Depicted in Figure 2 (With  $R = 5$  m,  $I = 2$  m, and  $V = 10$  m/s)

Actual TTP	$\tau_L^{(1)}$	$\delta\phi/\delta t$	$\tau_G$	$\delta\theta/\delta t$
5.0 s ( $Z = 50$ m)	5.14 s	0.44 $^\circ/s$	5.03 s	1.13 $^\circ/s$
4.0 s ( $Z = 40$ m)	4.18 s	0.67 $^\circ/s$	4.03 s	1.78 $^\circ/s$
3.0 s ( $Z = 30$ m)	3.25 s	1.13 $^\circ/s$	3.05 s	3.10 $^\circ/s$
2.0 s ( $Z = 20$ m)	2.39 s	2.19 $^\circ/s$	2.07 s	6.76 $^\circ/s$
1.0 s ( $Z = 10$ m)	2.11 s	4.00 $^\circ/s$	1.16 s	22.92 $^\circ/s$
0.7 s ( $Z = 7$ m)	4.27 s	2.21 $^\circ/s$	0.92 s	38.70 $^\circ/s$
~0.6 s ( $Z = 5.92$ m)	$\infty$	0.00 $^\circ/s$	0.85 s	46.98 $^\circ/s$
0.5 s ( $Z = 5$ m)	n/a	-3.09 $^\circ/s$	0.78 s	57.29 $^\circ/s$

system is only sensitive to the information available in the special case of image expansion.

In the present experiments, we examined the functional utility of global tau. Unlike other researchers investigating judgment of time to off-axis approaches, we specifically constructed our stimulus displays such that no local tau or other depth information (e.g., linear perspective) was available. In our first experiment, we examined observers' ability to make relative TTP judgments. In our second experiment, we assessed absolute judgment accuracy, both in the presence and absence of performance feedback.

### Experiment 1: Relative Judgments

Previous studies involving off-axis approaches (e.g., Schiff & Oldak, 1990) have used films of actual objects approaching in richly structured environments. Although such stimuli possess a good deal of ecological validity, the data collected in these studies have failed to demonstrate which of several sources of approach time information observers used. For example, in a filmed presentation, the image of a train will expand as it approaches (providing local tau information). It may demonstrate a particular edge rate for occluding evenly spaced objects in the scene (e.g., telephone poles along the track). Other pictorial and motion cues to approach time may also be available. Thus, in Experiment 1 we presented stimuli devoid of such extraneous distance cues to assess whether observers could make relative TTP judgments based on global tau. This relative TTP task required observers to identify which of two targets would transition their eye plane first, with the difference in TTP (i.e., the temporal separation of the targets) varying from trial to trial via a method of constant stimuli. We also examined whether the presence of one other motion cue partially specifying relative depth would influence judgments. That cue was the relative motion of the two targets when they were on the same side of the track vector. Two experiments were conducted: In the first, a small-screen display was used; in the second, a large-screen display was used to create compelling displays required for absolute TTP judgments.

#### Experiment 1A: Small-Screen Display

##### Method

**Observers.** Eight observers (6 male and 2 female) participated in this study. They ranged in age from 18 to 24 years; all had normal or corrected-to-normal vision. Prior to participation, each observer was informed of the general nature of the research.

**Apparatus and stimuli.** The experimental program was run on a Silicon Graphics Personal IRIS 4D/25TG workstation. Stimuli were displayed on a 1,280 × 1,024 pixel, 48.25-cm (diagonal) color monitor with a 60-Hz, noninterlaced raster refresh rate. Displays were updated at 60 frames/s. Observers viewed the screen binocularly through a reduction hood (i.e., a baffled viewing chamber), which maintained a fixed viewing distance of 76 cm and a field of view (FOV) of 19.2° horizontally and 16° vertically.

The stimulus displays consisted of a cloud of white, single-pixel dots ( $n = 600$ ). The eye point was translated through this volume at 1 graphical unit (gu) per frame, or 60 gu/s. The volume was 430

gu deep. Thus, during the 240 frames of each trial (4 s), the observer would traverse slightly more than half the volume. The viewing geometry was defined such that each graphical unit in software corresponded to 1 in. (2.54 cm) in virtual space. The track vector was centered on the display and perpendicular to the frontal plane (i.e., straight ahead). Two target dots, one green and one purple, were visible throughout the trial. Stimulus dots did not vary as a function of distance or undergo expansion as they approached. Thus, image expansion and size cues to depth were, by design, absent from the displays.

**Design.** A three-factor within-subject design was used. The first factor was motion information. For half the blocks of experimental trials, the two targets were on opposite sides of the track vector (tau-only condition); for the other blocks, they were on the same side (tau + relative motion condition). The second factor was the distance of targets from the track vector. Targets were displaced from the track vector by 5, 10, 15, or 20 gu, fully crossed. The final factor was the temporal separation of the target pairs. Pairs differed in depth location by 15, 30, 45, or 60 gu to create differences in TTP of 250, 500, 750, or 1,000 ms, with the far target 2.25–3.00 s from passage at the point of display termination. The distance from track vector created 16 combinations, crossed with four depth separations (64 cases), duplicated by reflection about the heading vector, for a total of 128 trials. A schematic of the viewing geometry and stimulus space for this experiment is shown in Figure 3.

Each block also contained 32 distractor trials (to equate the number of times the first target to pass was nearer or farther from the track vector in terms of visual angle) for a total of 160 trials per block. Following a practice block of 25 trials, observers completed four blocks of experimental trials (two with targets on opposite sides of the track vector and two with both targets on the same side). Observers made a binary response by placing a mouse-driven cursor into one of two boxes on the response screen and pushing the mouse button. The mouse button was then used to initiate the next trial. Feedback ("Response was correct" or "Response was incorrect") was given after each trial, in both the practice and the experimental blocks.

**Procedure.** Observers were instructed that viewing the displays would create a sense of moving through a star field composed of a large number of white dots and two colored (green and purple) dots. Their task was to judge which of the two targets would pass them (i.e., transition their eye plane) first, although the display terminated before either target reached them. Observers were instructed how to use the mouse to respond and initiate the next trial. They were allowed to rest between trials and were given a 10-min break between blocks. On average, 2 hr were required for an observer to complete the experiment.

##### Results

The percentages of correct responses averaged across observers and the targets' distance from track vector are shown in Figure 4A. There was a significant effect for motion condition (i.e., the presence vs. absence of relative motion),  $F(1, 7) = 6.84, p < .01$ , and a significant linear trend for temporal separation,  $F(1, 7) = 15.94, p < .01$ . There was no significant interaction between these two factors, indicating a purely additive effect. Performance was better than chance in all conditions except the 250-ms separation in the tau-only condition. The effect of target distance from track vector was assessed by conducting a linear trend analysis of variance (ANOVA) for the distance of each target from the track vector (5, 10, 15, or 20 gu). No significant effect on performance

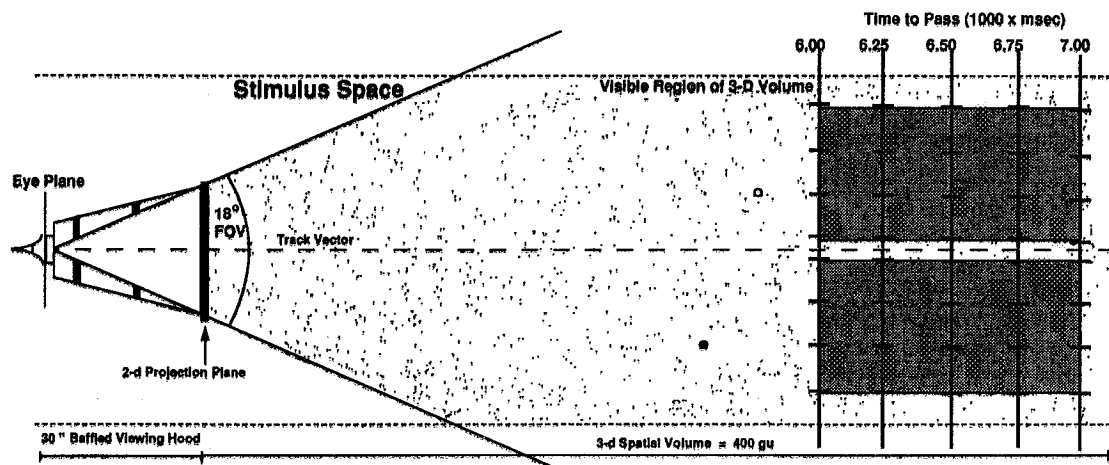


Figure 3. The viewing geometry and simulated stimulus space for Experiment 1A, viewed along the y-axis (i.e., overhead view). (FOV = field of view, gu = graphical units.)

was noted for distance from the heading vector for either the far target,  $F(1, 7) = 1.38$ , *ns*, or the near target,  $F(1, 7) = 0.96$ , *ns*. Also, regressions were conducted to assess whether the angular separation of the two targets affected performance. Separate regressions were run for each motion condition and temporal separation. None of these regressions were significant; the regression coefficients ranged from  $-0.21$  to  $0.10$ .

### Discussion

The results of Experiment 1A suggest that the observers were able to make reliable relative TTP judgments using global tau information. Performance was not affected by the distance of the targets to the track vector or by the angular distance between the targets. Performance was better in the tau + relative motion condition, suggesting that this second source of relative depth information added useful information.

Because the offset of the targets from the track vector varied, the reliability of the relative motion information is not

absolute. If the angular separation of the two targets decreases, it is always the case that the target closer to the track vector will pass first (the limiting case being that the closer target will occlude the farther one). However, an expansion of the angular separation is nondiagnostic; such motion can occur when the target nearer the heading vector is closer to the observer as well as when it is farther away from the observer.

Observers apparently applied a heuristic that maps contraction to the judgment that the target nearer the heading vector will pass first and expansion to the judgment that the target nearer the heading vector will pass second. The proportion of correct responses in the case of contraction was .95. The proportion of correct responses in the case of expansion when the target nearer to the heading vector was more distant was .85. The proportion of correct responses in the case of expansion when the target nearer the heading vector was closer was .60. In this last case, performance degraded with the magnitude of the expansion; the correlation between proportion correct and absolute angular expansion during the trial was  $-.85$ .

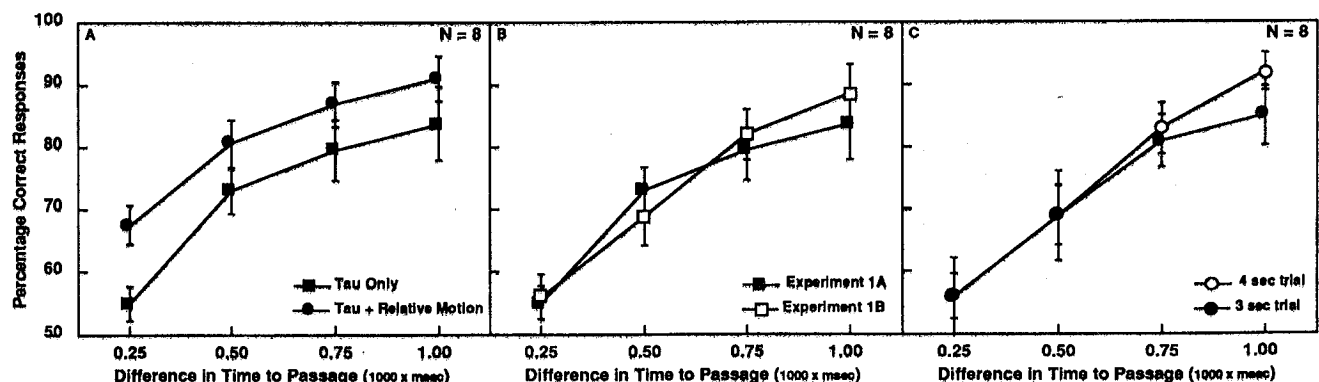


Figure 4. Percentage of correct responses (averaged across observers) for Experiment 1A's relative judgment task (A), the small-screen viewing condition (Experiment 1A) and the large-screen viewing condition (Experiment 1B) (B), and the 3- versus 4-s exposure trials in Experiment 1B (C).

Thus the relative motion cue was a partially reliable cue that observers used in addition to the global tau information. As Figure 4A and the analyses indicate, the combination of these two sources of relative depth information was additive. Even in the absence of relative motion, however, observers were able to make reliable relative TTP judgments for all but the shortest temporal difference.

### Experiment 1B: Large-Screen Display

In Experiment 1B, we examined the impact of a large-screen viewing condition. In Experiment 1A, we used a small-screen display viewed under a reduction hood. This display situation was insufficiently compelling for observers attempting to imagine the target continuing its trajectory toward and past their eye plane. Therefore, we changed to a large-screen display (46° monocular FOV at a viewing distance of 2.13 m) and modified the graphics program so that the clipping plane (i.e., the virtual volume of the display) terminated at the observer's eye instead of the screen plane. These changes created a much more compelling sense that the observer was traversing through the graphical space and made the task of estimating when a target would transition the eye plane much more intuitive. Finally, the depth of the viewing volume was increased to 684 gu to allow longer display sequences. The large-screen display layout is shown in Figure 5.

To assess the impact of these changes on the relative TTP judgments, we administered a relative-judgment task using the same stimulus space as the absolute-judgment task would employ. By using the same temporal differences between targets as in Experiment 1A (250, 500, 750, and 1,000 ms), we were able to compare relative-judgment performance under the two viewing conditions. We also used two lengths of stimulus duration (3 and 4 s) to determine whether the length of viewing time affected judgment accuracy.

### Method

**Observers.** There were 8 observers (four male and four female) who participated in both Experiments 1B and 2A. Observers ranged in age from 19 to 42 years; all had normal or corrected-to-normal binocular vision and were right-eye dominant.

**Apparatus and stimuli.** Displays were generated on the same graphics system used in Experiment 1A. The graphics program contained three modifications to accommodate the new display layout. First, the depth of the stimulus space was increased from 430 to 684 gu. Second, the FOV was increased from 19° to 46°. Finally, the clipping plane was moved from the screen plane to the observer's eye point. To replicate the tau-only condition of Experiment 1A, yet stay within the stimulus space for the absolute TTP task in Experiment 2, we created 20 unique dot pairs. The pairs were constructed to meet the following criteria: (a) They sampled the same stimulus space as the stimuli used in the absolute-judgment tasks (i.e.,  $x$  values between 25 and 76 gu and  $z$  values between 240 and 420 gu); (b) there were equal numbers of pairs representing temporal differences of 250, 500, 750, and 1,000 ms to allow comparison with Experiment 1A (tau-only condition) performance; and (c) trial duration could be 3 s or 4 s without either target exiting the FOV. This last criterion allowed us to examine the effect of stimulus duration on judgments. Displays terminated when the far target was 2–4 s from passage.

The display was projected on a  $2.44 \times 1.83$  m screen by an Electrohome RGB rear-projection system with 1,024-line resolution. As in Experiment 1A, a mouse was used to collect observers' responses and initiate the next trial.

**Design.** Each block consisted of 40 trials. Two blocks of trials were presented for both the 3-s and 4-s conditions, for a total of four blocks. A practice block of 25 trials was conducted before the experimental trials. The duration of the practice trials was the same as the first block of experimental trials. Thus, each subject practiced on either 3- or 4-s trials but not both.

**Procedure.** Observers were seated 2.13 m from the projection screen. They were fitted with a mask that occluded their left eye and reduced their right eye's FOV to a 46° circle. Observers were allowed to track the target dots if they desired, but were cautioned that tracking would reveal the edge of the display and therefore reduce the compelling three-dimensionality of the display.

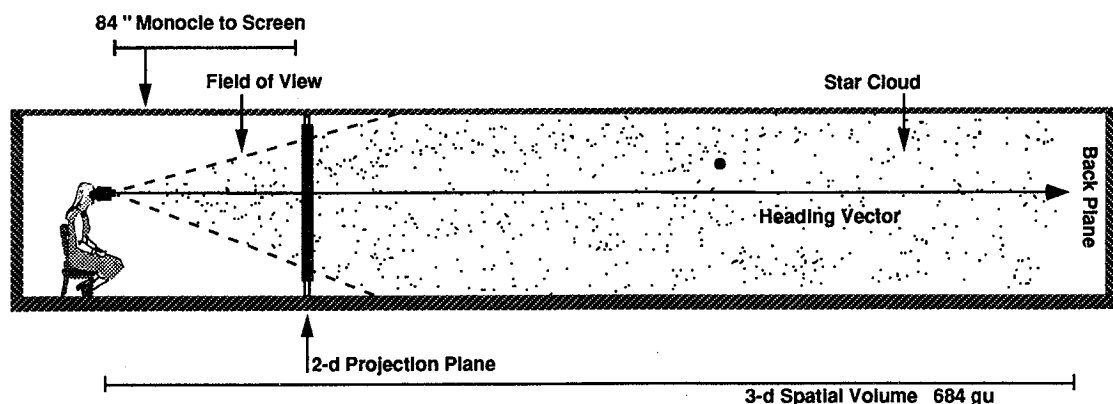


Figure 5. The large-screen display configuration used in Experiments 1B, 2A, and 2B. (The absolute-judgment time-to-passage configuration [i.e., one target] is shown. The relative-judgment configuration used two targets.)

Observers reported that they tended to keep their heads centered, because tracking one dot often resulted in losing sight of the other. Observers could rest between trials and were given 10-min breaks between blocks. As in Experiment 1A, feedback was given after each trial.

## Results

As can be seen in Figure 4B, observers' performance did not differ significantly from that demonstrated in the tau-only condition of Experiment 1A. Thus the changes in the viewing situation, necessary to enhance the compellingness of the display for the absolute-judgment task, did not affect relative judgments. Performance was also unaffected by whether observers viewed the targets for 3 or 4 s prior to display termination. As can be seen in Figure 4C, percentage of correct responses differed only for the longest temporal difference (1,000 ms); this difference was not significant,  $F(1, 15) = 1.05$ , *ns*.

## Discussion

As in Experiment 1A, observers were able to make reliable judgments of relative TTP when the difference in passage time was 500 ms or more. The changes in viewing conditions did not significantly enhance or degrade performance. Varying the exposure time from 3 to 4 s did not affect performance either. Given the constraints of our graphical space, it would be difficult to increase the exposure time much beyond 4 s while maintaining a reasonable distance to go at termination. However, it appears unlikely that additional exposure time would improve performance. Presumably, one could shorten exposure time and find a point at which performance starts to degrade, but examination of performance under such impoverished or reduced conditions was not the goal of these initial studies.

## Experiment 2: Absolute Judgments

Experiments 1A and 1B demonstrated that observers were able to make reliable relative TTP judgments when the difference in passage time was 500 ms or more. It is possible, however, that observers are sensitive only to the relative magnitudes of global tau. That is, observers could be able to judge which of two targets would pass first, yet be unable to extract the absolute temporal duration to either target's passage. Such a sensitivity to relative spatial distances without a corresponding capability to estimate absolute magnitudes is commonly found in the perceptual literature. However, this difference in relative and absolute distance judgment performance can often be attributed to the fact that the visual information available for judgments specifies depth only to a relative level (e.g., motion parallax). Global tau does, in theory, provide absolute temporal distance information. In Experiment 2, we examined observers' ability to use this information.

### Experiment 2A: Absolute Judgments With Feedback

#### Method

**Observers.** The same 8 observers (4 male and 4 female) who participated in Experiment 1B participated in this experiment.

**Apparatus and stimuli.** The displays were the same as those in Experiment 1B, with the following modifications: (a) Only one colored target was displayed, and (b) no response menu appeared; instead, the observers used the mouse button to indicate when they thought the target would transition their eye plane.

**Design.** Target dots were placed within the trapezoidal stimulus spaces shown in Figure 6. Targets were between 1 and 3 s to passage when they exited the FOV at  $\pm 23^\circ$ . By sampling within the space shown, we made TTP fully independent of the time the target was visible on screen and largely independent of its initial angular distance from the heading vector. Sixty-six trials were placed within this space with 6 gu spacing in the *x* dimension and 12 gu (i.e., 200

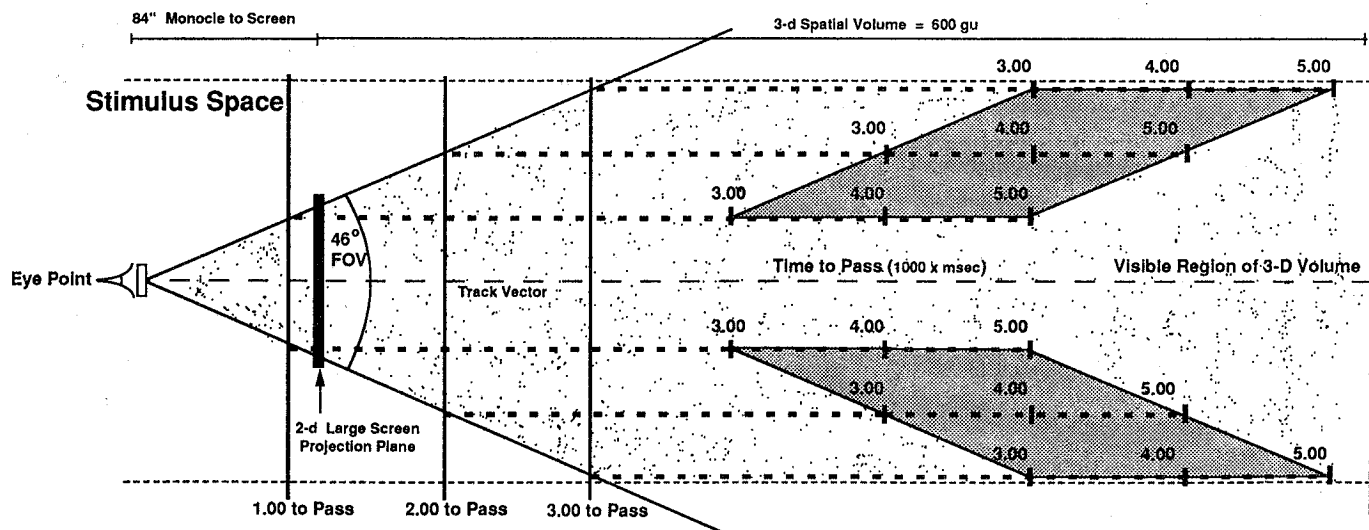


Figure 6. Viewing geometry and simulated stimulus space for Experiment 1B. (As in Figure 3, the *y*-axis [i.e., overhead view] is depicted.)

ms) spacing in the  $z$  dimension. After a practice block of 25 trials, observers completed three blocks of trials. They were given 10-min breaks between blocks.

**Procedure.** The single target appeared to the right or left of the heading vector. Observers were informed that they would overtake and pass this dot, but that it would go out of their FOV before that. Their task was to imagine that the target continued its trajectory and to press the mouse button when they thought the target would transition their eye plane. Observers were instructed not to move their heads to track the target. The target was visible for 3–5 s, and the flow field continued after the target left the FOV until the observer pressed the mouse button. Following each trial, observers were given feedback on how early or late their judgment was; the error was printed on the screen in milliseconds.

## Results

Figure 7 shows the scatterplots of actual TTP versus judged TTP for the 8 observers. The TTP is plotted in terms of time from the target's appearance, that is, display time (3–5 s) plus extrapolation time (1–3 s). The linear fits ( $R^2$ ) for the data ranged from .55 to .84, with a mean of .73. The regression slopes for all observers were less than 1, indicating a temporal compression (i.e., an additional second in actual TTP resulted in a less than 1-s increase in judged time). The intercepts were all positive. This, coupled with the less-than-unity slopes, indicates that shorter TTPs were overestimated and longer TTPs were underestimated. Across observers, the correlation between constant error and extrapolation time was  $-.94$ .

Because right-hemisphere dominance has been demonstrated for several aspects of spatial perception, including depth perception based on disparity (Durnford & Kimura, 1971; Kimura, 1973), we examined whether performance varied as a function of visual field. To this end, a second set of regression analyses was conducted with each observer's data divided into nasal and temporal projection sets. Because all observers viewed the displays monocularly with their right eye, nasal projections mapped to the left visual field (processed by the right hemisphere). Temporal projections mapped to the right visual field and were processed by the left hemisphere. The regression equations for these two sets did not differ significantly in terms of fit (mean  $R^2$  was 0.77 for temporal projection and 0.73 for nasal projection), slope (mean slope was 0.86 for temporal and 0.79 for nasal), or intercept (mean intercept was 0.83 for temporal and 1.02 for nasal).

### *Experiment 2B: Absolute Judgments Without Feedback*

Because the target always exited at the same eccentricity, it is possible that the strong correlation between actual and judged TTP in Experiment 2A was merely the result of observers' using the feedback to calibrate their responses on the basis of the target's terminal image velocity. That is, given that the observers' velocity through the depicted space was constant over trials (it was, at 60 gu/s), an observer could have achieved impressive performance on this task by noting a monotonic (albeit nonlinear) relation between the target's

image velocity as it exited the field of view and the TTP. The feedback we provided in Experiment 2A could, in theory, have allowed observers to calibrate this function. Thus, we replicated the conditions of Experiment 2A, with the exception that observers were not given feedback on the accuracy of their judgments.

## Method

**Observers.** Four observers (3 male and 1 female) participated in the study. All had normal or corrected-to-normal vision and ranged in age from 24 to 34 years. None had participated in the earlier experiments.

**Apparatus and stimuli.** The same stimuli and viewing conditions used in Experiment 2A were used in this experiment.

**Design.** As in Experiment 2A, each block consisted of 66 trials, varying the target's displacement from the heading vector (25–76 gu) and initial depth (240–480 gu). Again, the target exited observers' FOV at  $23^\circ$  to the left or right of the FOE.

**Procedure.** Following two demonstration trials, in which they neither gave judgments nor received feedback, observers completed three blocks of trials without feedback. After a 10-min break, they completed three blocks of trials with feedback.

## Results

Figure 8 shows the scatterplots of actual TTP versus judged TTP for the 4 observers in the absence and presence of feedback. Across observers, the presence of feedback did not significantly affect the linear regression fits, in terms of either slope and intercept or amount of variance accounted for (i.e.,  $R^2$ ). The regression equations were similar to those noted in Experiment 2A; six of the eight fits had slopes less than 1 and positive intercepts. The range of squared multiple correlation values was also similar (0.66–0.85).

## Discussion

In both Experiments 2A and 2B, observers were able to make reasonably accurate absolute TTP judgments. They were able to make such judgments even in the absence of feedback. This suggests that they were not simply basing their judgments on the target's terminal image velocity (calibrated via feedback), because judgments were equally good in the absence of feedback. This would suggest that observers can actually extract absolute temporal range from global tau information.

Observers did, however, demonstrate a systematic tendency to overestimate shorter TTPs and underestimate longer TTPs. The properties of observers' response curves (positive intercept and slope less than 1) resemble those noted in other studies of absolute time-to-arrival estimates (e.g., McLeod & Ross, 1983; Schiff & Detwiler, 1979; Schiff & Oldak, 1990). These response patterns emerge over a variety of approach conditions (i.e., direct and off axis), stimulus targets, background contexts, and presentation procedure. As other researchers have suggested (Schiff & Oldak, 1990), these biases could either result from a distortion (or warping) of the visual/temporal space or reflect an artifact introduced by the cognitive extrapolation required of observers. (The pattern of



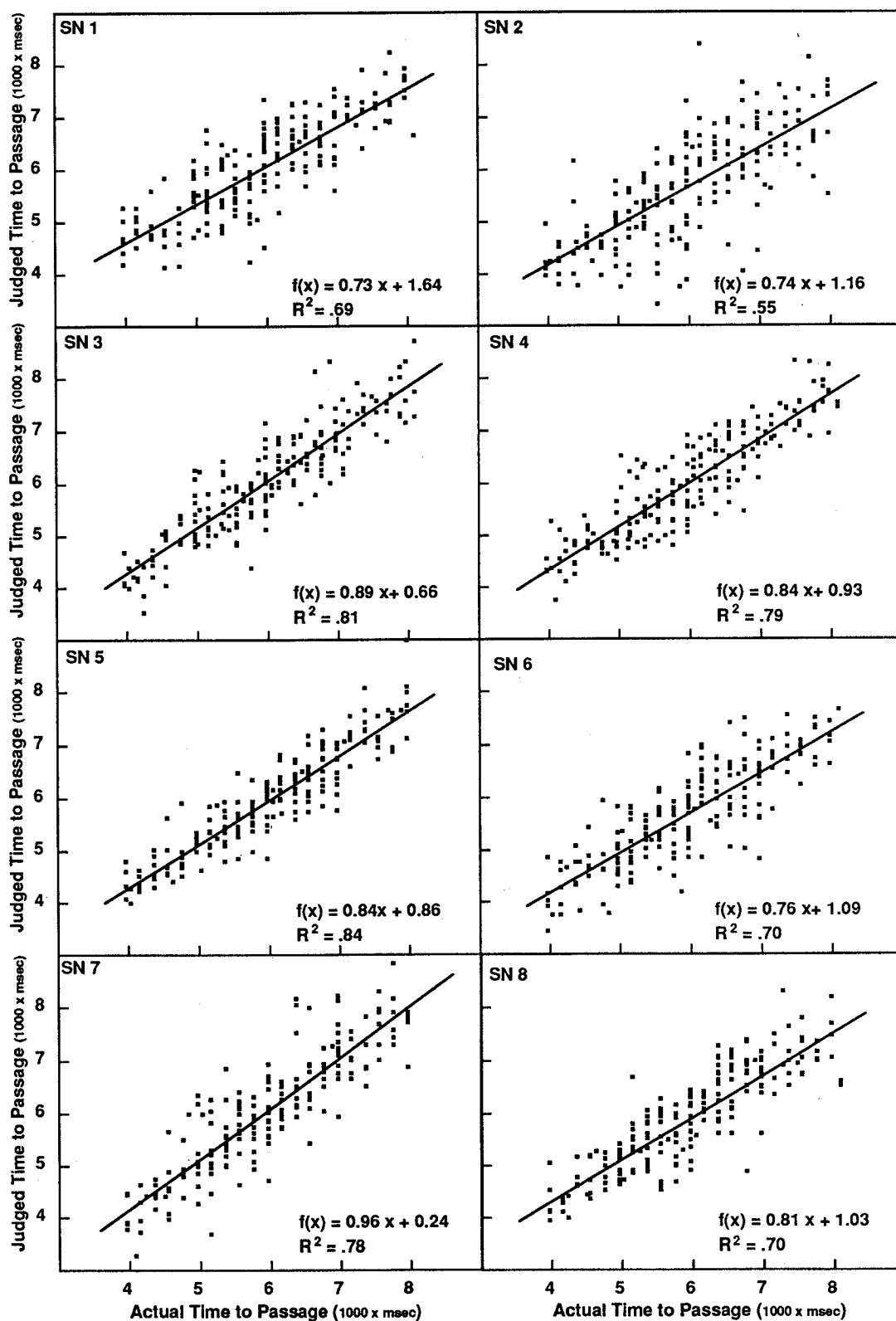


Figure 7. Scattergrams of the 8 observers' absolute time-to-passage (TTP) judgments from Experiment 2A (actual TTP vs. judged TTP). (Regression equations and fits are shown for each observer.)

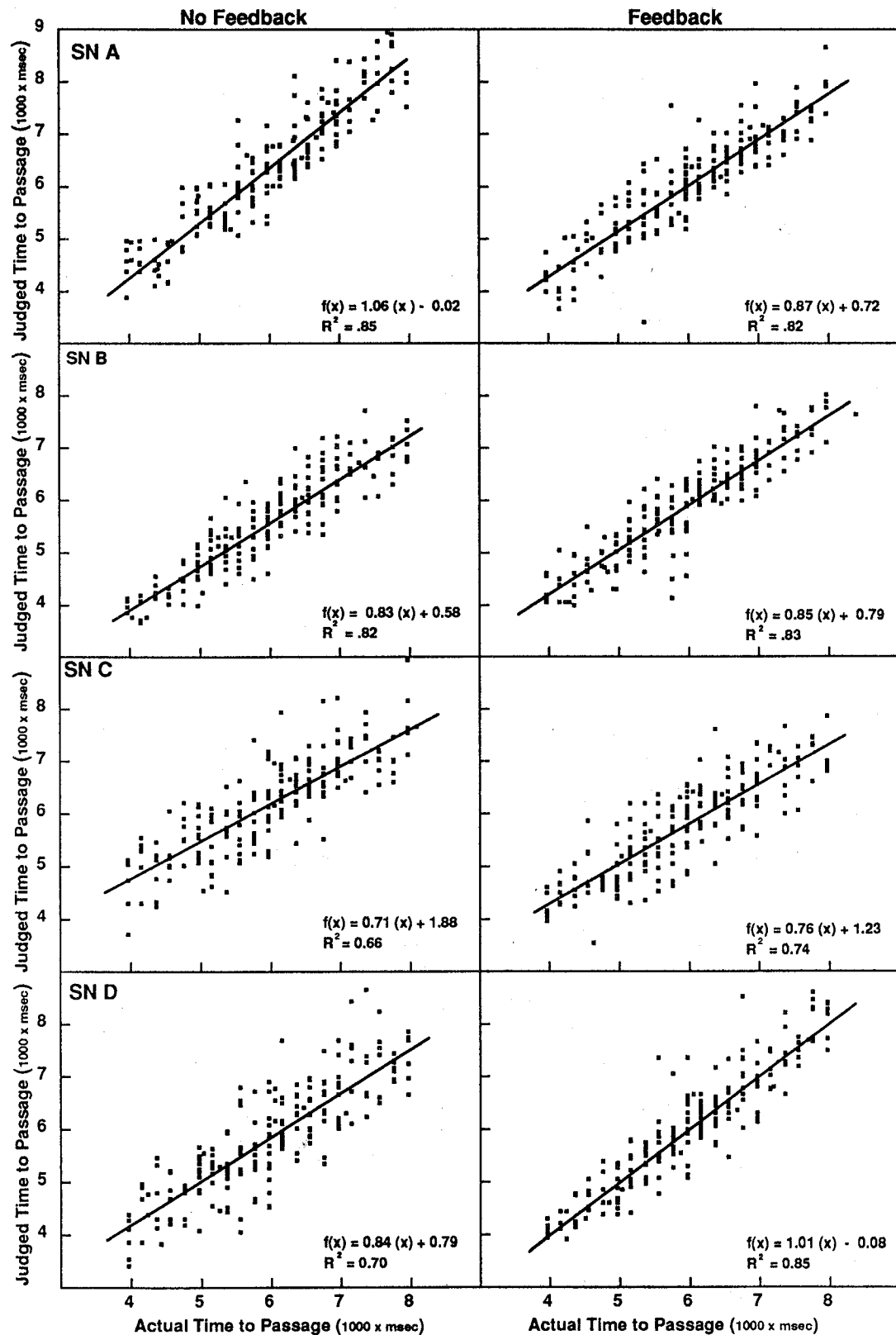


Figure 8. Scattergrams of the 4 observers' judged time-to-passage (TTP) versus actual TTP in the absence and presence of feedback from Experiment 2B. (Regression equations and fits are shown for each data set.)

errors is certainly consistent with pure temporal extrapolation tasks.) Further research is required to determine which factor(s) account for the pattern of errors.

### General Discussion

In the present experiments, observers demonstrated accurate and robust use of global tau in both relative and absolute TTP judgment tasks. Relative TTP judgments were reliable when the difference in passage time was a half second or more. In studies of observers' relative TTC judgments, reliable performance has been found at smaller target separations. Todd (1981) reported more than 90% accuracy for differences as small as 150 ms and above-chance performance for differences of more than 10 ms for direct approach stimuli terminated 3 s from impact. Regan and Hamstra (1992) reported a discrimination threshold of 7–13%, constant over a range of TTCs of 1–4 s (this maps to 210–390 ms for Todd's 3-s termination condition). Taken together, these findings suggest a somewhat greater sensitivity to relative TTC than relative TTP, at least for events that terminate fairly close to the observer (i.e., at a temporal range of 4 s or shorter). This greater sensitivity for TTC information would be consistent with both the different functional roles of the two information sources (TTC being used for precise sensorimotor skills such as catching and TTP being used for more global navigation) and the relative complexity of their underlying optical variables (TTC requires only local image processing, but TTP requires integration across the global flow field). These differences in sensitivity might also reflect differing neural bases for their detection. Future research should address whether this advantage for TTC information diminishes (or, in fact, reverses) for longer temporal windows, if only by virtue of the TTC information's becoming subthreshold.

Performance did not vary as a function of the distance between the targets or the offset of the target from the track vector. This finding stands in contrast to the performance of most passive-ranging systems (e.g., Sridhar, Suorsa, & Husien, 1993). In those systems, the precision of range estimates degrades as the offset of the target from the track vector decreases. This results from the fact that the angular velocity of an object at a given range is a function of the offset, and small velocities are difficult to derive precisely from sensor data. Human observers likewise have problems judging small object motions, but this limitation is probably offset by limitations in estimating the angle between the target object and the track vector. Thus, at a given range, an object proximal to the track vector will have a small angular velocity but also a small angle between it and the track vector; an object with a larger offset will have a greater angular velocity but also a greater angular distance to the track vector. Unlike mechanical systems, whose precision of angular distance estimation is independent of magnitude, human observers likely find the error in these two estimation tasks offsetting and hence demonstrate consistent performance.

The absolute TTP findings in the present experiments demonstrate that performance was also robust across visual field (i.e., nasal vs. temporal projection to observers' right eyes).

In addition to providing information about basic visual performance capabilities (in this case, homogeneity across the visual field), such findings have important implications for human factors applications, because several sensor systems (e.g., the Forward Looking Infrared system with which military pilots fly helicopters) use monocular displays, and pilots' head motions are constrained.

Observers in the absolute judgment task did demonstrate nonveridical temporal scaling, with slopes less than unity and positive intercepts. This means that shorter TTPs were overestimated and longer TTPs were underestimated. It is not clear whether this bias represents a warping of the perceptual space (e.g., a target 4 s distant appears less than twice as far as a target 2 s distant) or results from systematic error in the cognitive extrapolation component of the judgment task. Further research is needed to decompose this bias into its components. Despite this bias, however, observers' judged TTPs were highly correlated with actual TTPs. Furthermore, observers did not require any training or feedback to achieve well-calibrated judgments. This suggests that global tau does provide a useful source of temporal information.

Given that people are likely to use global tau to orchestrate control and avoidance maneuvers, it is interesting to consider a degenerative case of global tau that occurs when the observer and a moving object are on a collision course but the object is not on the observer's track vector. If the observer and object maintain constant velocities, the center of the object maintains a fixed angle relative to the observer's track vector (see Figure 9). Thus,  $\theta$  for the centroid of the object is constant (i.e.,  $\delta\theta/\delta t$  is zero), and  $\delta\theta/\delta t$  for all other points is small, reflecting only image expansion. Consider what value of global tau is specified in this condition: Because  $\tau_G = \theta/\delta\theta/\delta t$ , as  $\delta\theta/\delta t$  approaches zero, global tau approaches infinity. Thus, an object on such a collision course can be mistaken for an object at a very large distance, because the global tau information is virtually identical. Local tau information will be veridical, but it may not be salient at greater distances. Only when the image expansion becomes salient (or if the observer is cued by some nonmotion information,

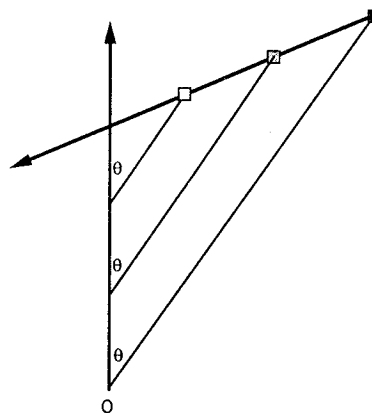


Figure 9. An observer and an object are on trajectories that will result in a collision. ( $\theta$  is the angle between the observer's track vector and the object's centroid and remains constant so long as the two bodies maintain constant velocities.)

such as familiar size) are the two cases discriminable. Because image expansion may not become salient until the object is temporally proximal, the observer may be required to make a last-second correction to avoid collision. Such incidents are highly undesirable in flight situations, but an examination of the global tau information sheds light on how such mishaps may occur despite the visual scan strategy pilots are instructed to use, which includes "looking for objects that aren't moving."

This "unmoving objects on a collision course" scenario, however, represents a degenerate (albeit interesting) case of global tau information. Most of the time, global tau provides reliable information concerning objects' temporal distance. Moreover, observers demonstrate a robust ability to use this information. Thus, we propose that global tau should be added to the catalogue of dynamic and static visual cues that provide people with a rich source of visual information to use in planning and orchestrating their movements through and interactions with the environment. Further research is needed to understand how these sources are integrated and combined (DeLucia, 1991).

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### Appendix

#### The Geometry of Time-to-Passage ( $\tau_G$ ) and Its Relation to Time-to-Contact ( $\tau_L$ )

The geometry for the passage event is shown in Figure 1. Our derivation is similar to Lee (1976) except that we assume a moving observer and stationary object (instead of a stationary observer and moving object).

The observer,  $O$ , is moving with velocity,  $V$ , along a track vector. An object,  $P$ , lies some distance,  $R$ , from this track vector, such that the angle between the track vector and  $P$  is  $\theta$ . As the observer moves forward, the distance ( $Z_t$ ) between the observer and the passage

plane (i.e., the plane orthogonal to the heading vector on which  $P$  lies) decreases, thereby increasing the angle  $\theta$ .

Using the law of small angles, at any time  $t$ :

$$\theta_t = \arctan(R/Z_t) \approx R/Z_t, \quad (A1)$$

Define the velocity  $V$  at time  $t$ :

$$V = -\delta Z/\delta t. \quad (A2)$$

(The negative sign keeps velocity positive.) Because, at time  $t$ ,  $\theta = R/Z$ ,

$$\delta\theta/\delta t = -[R \cdot \delta Z/\delta t]/Z^2 = R \cdot V/Z^2. \quad (A3)$$

Thus, by substitution,

$$V = [-\delta Z/\delta t \cdot Z^2]/R = [\delta\theta/\delta t \cdot R]/\theta^2. \quad (A4)$$

At any time  $t$ , time-to-passage (TTP) is physically defined as

$$TTP = Z/V. \quad (A5)$$

Substitution allows us to express TTP in purely optical variables:

$$TTP = Z/V = (R/\theta)/[(\delta\theta/\delta t \cdot R)/\theta^2] = \theta/(\delta\theta/\delta t) = \tau_G. \quad (A6)$$

Mathematically, an equivalent derivation can be performed for time to contact (TTC), or  $\tau_L$ . The only difference is that rather than

the angle between the object and the track vector, the angular extent between two points on an object ( $\phi$ ) is used, where  $\phi$  is approximated by the distance between those points,  $I$ , divided by the distance,  $Z$ , between the observer and the object. Thus, at any time  $t$ ,

$$TTC = Z/V = \phi/(\delta\phi/\delta t) = \tau_L. \quad (A7)$$

Note that for off-axis approaches, the relationship  $\phi = I/Z$  used to derive  $\tau_L$  is no longer valid. Instead,

$$\phi = \cos\theta \cdot I/(Z^2 + R^2)^{1/2}, \quad (A8)$$

where  $\phi$ ,  $\cos\theta$ , and  $Z$  are all time varying. Thus  $\tau_G$  is appropriate for off-axis approaches.

Received June 15, 1992

Revision received October 19, 1992

Accepted October 28, 1992 ■

